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PART I: ANALYSIS

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# WAVE PROPAGATION IN A DC SUPERCONDUCTING CABLE Part I: Analysis\*

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Abstract - We consider a dc superconducting cable design consisting of four concentric metallic cylinders, of which two carry the load current and two comprise the cryogenic enclosure. When a transient voltage is impressed across such a cable, the major dielectric may not be fully stressed to its design value, and unwanted voltage stresses may develop across other parts of the cable. This paper analyzes the surge-voltage propagation characteristics of a four-conductor dc superconducting cable for a step-function input voltage. This analysis, although mainly directed to superconducting cables, is also applicable to other multiconductor transmission lines. A companion paper discusses the parametric effects of the cable system in optimizing the voltage distribution.

#### INTRODUCTION

The advantages of transmitting bulk power by a dc superconducting cable has been described previously [1]. However, such a cable must operate reliably under system constraints, e.g., harmonics on the dc, system faults, and transient overvoltages. The behavior of a dc superconducting cable, which is being developed at the Los Alamos Scientific Laboratory (LASL), under dc harmonics and system faults has been reported in an earlier paper [2]. This paper discusses the theory of wave propagation in a multiconductor cable system and describes the response of a dc superconducting cable of a specific design to transient voltages. The effects of parametric changes in the system as well as in the cable design, including those of dc conventional cables, are discussed in a companion paper [3].

Figure 1 shows the dc superconducting cable of LASL design. It consists of four concentric cylinders. The innermost cylinder (conductor 1) carries the load current and consists of subcables made up of wires of multifilamentary Nb3Sn superconductor embedded in copper matrix. The second cylinder (conductor 2), which carries the return current, also consists of copper-stabilized multifilamentary Nb3Sn superconductor. The cryogenic enclosure is composed of the tnird and fourth concentric cylinders. The inner cylinder (conductor 3) of the cryogenic enclosure is made of stainless steel and the outer cylinder (conductor 4) of carbon steel. The space between conductors 1 and 2 is filled with wrapped tape dielectric

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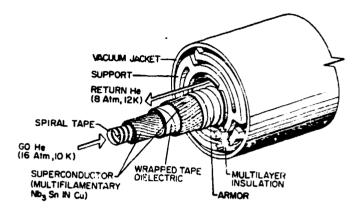


Fig. 1. Conceptual design of a four-conductor dc superconducting cable.

to withstand the system voltage - steady-state dc and transient overvoltages. The space between conductors 2 and 3 is filled with supercritical helium at 1.33 MPa and 12 K. The space between conductors 3 and 4 is evacuated and filled with multilayer thermal insulation. Conductor 4 is isolated from earth by a thin layer of insulation.

When a transient appears on conductor 1, multivelocity voltage waves will propagate along the four conductors and create voltage differences across each pair of conductors, caused by the disparity in the wave velocities that did not exist initially [4]. A flashover or puncture may occur between any pair of conductors if the voltage build-up across the conductor pair exceeds the dielectric strength of the space between them. The weak member of the system may be the evacuated space filled with multilayer thermal insulation within the cryogenic enclosure. Repeated punctures within this space would increase heat leak to the cable, reducing its efficiency.

#### METHOD OF ANALYSIS

A high-power dc superconducting cable will most likely transmit power from an energy park to an urban load center, or to feed power to a metropolis from rural overhead lines. Such a cable will be long (50 km or longer). Although the return-current conductor (conductor 2) and the cryogenic enclosure (conductors 3 and 4) are nominally at ground potential for steadystate operation, this assumption may not be valid under transient conditions. To inhibit stray current from flowing through these "grounded" conductors, ore end of each of these conductors will be kept isolated from ground either by keeping one terminal open or connecting it to earth through a capacitor or through a surge arrester. Furthermore, even when one terminal of each of these three conductors is grounded, there will always be a finite grounding resistance between the conductor and the earth. Therefore, the analysis should include the resistance of each conductor and the terminal grounding conditions.

The analysis becomes simple if it is assumed that the earth is perfect, i.e., an earth of infinite electrical conductivity. In practice this ideal condition is never met. The problem becomes more complex if the long cable is laid not only in regions of divergent earth resistivities but also in layers of earth of different resistivities. Moreover, lack of precise knowledge of the earth resistivity precludes exact numerical computations. In spite of these difficulties, it is worthwhile to study the effect of imperfect earth in order to assess the manner in which attenuation and distortion of voltage waves in a cable takes place.

## Rudenburg-Hayashi Model of Transient Impedances

Transient phenomena, such as switching and lightning surges, are usually analyzed using the Laplace transform. Therefore, it is desirable to express the transient impedances in terms of the Laplace-transform operator s.

Rudenberg's analysis [5] of imperfect earth has been extended by Hayashi [6] so that the earth-correction terms can be expressed in terms of the operator s. Rudenberg has represented an underground cable by laying its center conductor on the ground level at the center of a groove of a semicircular shape of radius h from which the earth has been scooped out, h being the distance of the center conductor of the original cable from earth (Fig. 2). The ground impedance as a function of the Laplace-transform operator s can then be expressed as

$$Z_g(s) \simeq \delta_{g1} s^{1/2} - \delta_{g2} + \delta_{g3} s^{-1/2} - \dots$$
 (1)

For fast transients, only the first two terms are important, where

$$\delta g_1 = \sqrt{10 \text{ Rg}} \times 10^{-4}$$
  
 $\delta g_2 = Rg/4$   
 $R_g = 2 \rho_g / h^2$   
 $\rho g = \text{soil resistivity}, \Omega - m$   
 $h = \text{distance of center conductor of cable from ground}, m$ 

Similarly, the transient impedance of the cable conductor can be expressed as

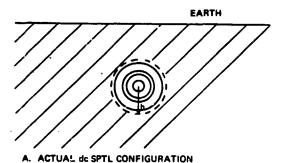
$$Z_c(s) = \delta_{c1} s^{1/2} + \delta_{c2} + \dots$$
 (2)

where

$$\delta_{c1} = \sqrt{10 R_c} \times 10^{-4}$$
  
 $\delta_{c2} = R_c/4$   
 $R_c = dc$  resistance of the conductor.

For a cable consisting of n concentric cylinders, the earth-correction impedance matrix will then be

$$[\delta] \propto s^{1/2} [\delta_1] + [\delta_2],$$
 (3)



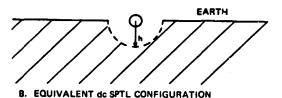


Fig. 2. Rudenberg's model of cable in imperfect earth.

# Solution of Transmission-Line Equations

The Laplace transform of the transmission-line voltage equations can be written in matrix form as

$$\partial^{2}[V]/\partial x^{2} = (s^{2}[L][C] + s^{3/2}[\delta_{1}][C] + s[\delta_{2}][C]) [V]$$

$$= [0]^{2}[V]$$
(4)

where

$$[Q]^2 = (s^2[L][C] + s^{4/2} [\delta_1][C] + s[\delta_2][C])$$
,

[L] and [C] are the inductance and capacitance matrices of the n-conductor cable, and

[ $\delta_1$ ] and [ $\delta_2$ ] are given by Eq. (3).

The solution of Eq. (4) is given by

 $[V_0] = [V] \text{ at } x = 0.$ 

$$[\overline{V}] = \exp(-[Q]x) [\overline{V}_{0}]$$

$$= \exp(-s[M]^{\frac{1}{2}}) \exp(-\sqrt{s} [B_{1}]x/2) \exp(-[B_{2}]x/2)$$
where
$$[\overline{V}_{0}], \qquad (5)$$

$$[M] = [L][C], \qquad [B_{1}] = [M]^{\frac{1}{2}} [M]^{-1} [\delta_{1}][C], \qquad [B_{2}] = [M]^{\frac{1}{2}} [M]^{-1} [\delta_{2}][C],$$

Each of the three exponential terms needs to be expanded by Sylvester's expansion theorem [6], giving

$$\exp(-s[M]_x^{1/2}) = \sum_{r=1}^{1/2} \exp(-sq_{1r}x) [a_{1r}]$$
 (6)

$$\exp(-\sqrt{s}[B_1]x/2) = \sum_{r=1}^{n_2} \exp(-\sqrt{s} q_{2r}x/2)[a_{2r}]$$
 (7)

$$\exp(-[B_2]x/2) = \sum_{r=1}^{n_3} \exp(-q_{3r}/2) [a_{3r}],$$
 (8)

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where 
$$[a_{1r}] = \frac{p=1, \dots, n_1}{p + r} \left( \frac{q_{1p}^2[U] - [M]}{q_{1p}^2 - q_{1r}^2} \right)^{d_{1p}} .$$

$$[a_{2r}] = \frac{p=1, \dots, n_2}{p + r} \left( \frac{q_{2p}[U] - [M]}{q_{2p}^2 - q_{2r}^2} \right)^{d_{2p}} .$$

$$[a_{3r}] = \frac{p=1, \dots, n_3}{p + r} \left( \frac{q_{3p}[U] - [M]}{q_{3p}^2 - q_{3r}^2} \right)^{d_{3p}} .$$

 $q_{1r}$  (r = 1,...n) are distinct eigenvalues of [M],  $q_{2r}$  and  $q_{3r}$  are distinct eigenvalues of [B<sub>1</sub>] and [B<sub>2</sub>] respectively, [U] = identify matrix,  $q_{1p}$  = order of degeneracy of  $q_{1p}$  eigenvalue (i = 1,2,3), and  $d_{11}+...+d_{n1} = d_{21}+...+d_{n2} = d_{31}+...+d_{n3} = n$ 

#### **Boundary Conditions**

The length of the cable has been assumed to be semi-infinite in this study; therefore, there would be no reflections from the far end. A voltage V<sub>01</sub> is injected into conductor 1 at x=0, while the other conductors are either grounded through a grounding resistance Rg or left open. In this case, Voz...V need to be evaluated. These values can be obtained, for known grounding resistances, by solving for the currents at x=0.

The Laplace transform of the current matrix is

where [V] is obtained from Eqs. (5)-(8).

#### Final Solution

The closed-form solution for the voltage waves, in time domain, is shown in the following equation.

$$[V(t)] = \left\{ \sum_{k=1}^{n_{1}} [a_{1k}] \sum_{\ell=1}^{n_{2}} [a_{2\ell}] \operatorname{erfc}(q_{2\ell}^{x/4\sqrt{t-q_{1}K^{x}}}) \times \left\{ \sum_{m=1}^{n_{3}} [a_{3m}] \exp(-q_{3m}^{x/2}) \right\} [V_{Q}(t)] \right\}$$
where

 $\mathbf{u}(\mathbf{t}-\mathbf{q}_{\mathbf{1}\mathbf{K}^{\mathbf{x}}})$  - Delayed unit function.

# Cable Parameters

Computation of the voltage waves requires that the resistance matrix  $[\delta]$ , the inductance matrix [L], and the capacitance matrix [C] of the cable be known. As discussed before, the  $[\delta]$  -matrix was computed by Hayashi's method.

The [L]-matrix was evaluated by computing Maxwell's electromagnetic coefficients from the following set of equations,

$$\phi_{r} = L_{r1}i_{1} + \dots + L_{rs}i_{s} + \dots + L_{rn}i_{n}$$
, (11)

where

Lrs - Lsr,

 $\phi_{r'}$  = total magnetic flux linking conductor r,  $\frac{1}{r}$  = current flowing through conductor r, and r = 1,...., n.

The transient current was assumed to flow along the outer surface of the conductor. The permeability  $(\mu)$  of a magnetic material was assumed equal to that  $(\mu_0)$  of free space when a steep-front transient is applied. The coefficients of inductance  $(L_{rs})$  are then given

$$L_{rs} = (\mu_0/2\pi) \ln (h/r_{s0}) \text{ for } r \le s, H/m,$$
 (12)

 $μ = 4π \times 10^{-7}$  H/m,  $h^0$  = distance of earth from center of cable, in  $r_{so}$  = outer radius of conductor s, m.

The [C]-matrix was evaluated by computing Maxwell's electrostatic coefficients from the following set of equations,

$$Q_r = P_{rl}e_l^+ \dots + P_{rs}e_s^+ \dots + P_{rn}e_n^-$$
 (13)  
 $P_{rs} = P_{sr}^-$ 

where

$$Q_r$$
 = Electrostatic charge on conductor, r,  $e_r$  = Potential of conductor r, and r = 1, ...., n.

The coefficients prs are then given by

$$p_{rn} = \ln(h/r_{n0})/2\pi\epsilon_n$$
 for  $r \le n$  (14)

$$P_{r(n-1)} = P_{nn} + \ln (r_{n1}/r_{(n-1)0})/2\pi\epsilon_{(n-1)}$$
 for  $r < (n-1)$ 

where

The [C]-matrix is then given by

$$[c] - [p]^{-1}$$
 (15)

#### COMPUTATION OF VOLTAGE WAVES

# Cable and System Parameters

The conceptual design of the cable is shown in Fig. 1. Table I shows the pertinent parameters of the cable. In the analysis, the transient current was assumed to be expelled from the superconductor into the stabilizing copper. The conductors were assumed to be perfect concentric cylinders, and the cable semi-infinite in length.

The unit step-function voltage wave was applied to conductor 1 at #=0 while each of the other three conductors were connected to earth through a grounding resistance of 10  $\Omega$  . The earth resistivity was assumed to be 100  $\Omega$ -m.

A companion paper shows the effects of various values of grounding resistance, earth resistivity, and cable design parameters on the propagation characteristics of the voltage waves [3].

#### Computation

A numerical Fortran computer program was developed to solve the voltage equations for a given cable

#### TABLE I

Parameters of 100-kV dc Superconducting Cable

Outer radius of conductor 1,	r10	=	22.7 mm
DC resistance of conductor 1,	R	•	0.2 μΩ/m
Inner radius of conductor 2,			28.3 mm
Outer radius of conductor 2,			35.0 mm
DC resistance of conductor 2,			0.2 μΩ/m
Inner radius of conductor 3,			45.0 mm
Outer radius of conductor 3,	r <sub>30</sub>		49.0 mm
DC resistance of conductor 3,	R_3	-	<b>440.0</b> μΩ/m
Inner radius of conductor 4,			90.0 mm
Outer radius of conductor 4,			94.0 mm
DC resistance of conductor 4,			43.0 μΩ/m
Distance of cable center to eart	•		
Dielectric constants:	k <sub>1</sub>	-	2.2
	k <sub>2</sub>	•	1.0
	_		1.0
	٠,		3.0

configuration. The voltage waves on each conductor of the cable, as per unit of the applied voltage, are computed at time and distance coordinates specified by the user.

The data put into the program are the number of conductors in the cable, their dc resistances and radii, the dielectric constants of the materials between the conductors, the known initial voltages, the distances between the center of conductors and earth, the earth resistivity, and the terminal resistances between the conductors and earth at x=0.

The code includes a graphics package which plots two-dimensional graphs of the wavefronts at specified distances.

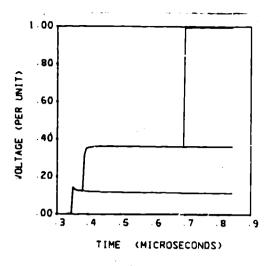
Figure 3 shows the voltage waves on each of the four conductors of the cable at distances of 0.1, 1, and 10 km from the origin.

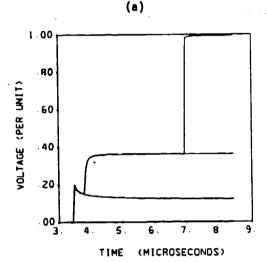
#### DISCUSSION

## Interpretation of Data

With reference to Fig. 3, the original wave is split into four component waves traveling at unequal velocities. All four component waves are present on conductor 1. The number of component waves decreases progressively from conductor 1 to conductor 4. One single voltage wave of small magnitude ravels on conductor 4 (nearest to earth). This component wave which is present on all four conductors at x=0 is highly attenuated within a short distance from the origin. This wave is not discernable in the figures.

The voltage between each conductor and ground changes abruptly as the component voltage waves arrive at a point along the cable at different times. For instance, conductor 1 experiences abrupt voltage changes at 3.45, 3.86, 5.77 and 6.93  $\mu$ s as the four component waves arrive at x=1 km (Fig. 3b). Similarly, thre' component waves arrive on conductor 2 at 3.45, 3.86, and 5.77  $\mu$ s; two component waves on conductor 3 at 3.45 and 3.86  $\mu$ s; and, one single wave on conductor 4 at 5.77  $\mu$ s. The component wave (of small magnitude) which arrives at x=1 km on all four





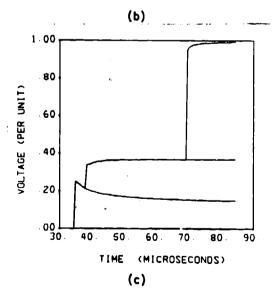


Fig. 3. Step response of a scmi-infinite 100-kV four-conductor dc superconducting cable.  $R_g=10~\Omega$  at x=0;  $\rho_g=100~\Omega$ -m. (a) x=100 m; (b) x=1 km; (c) x=10 km. Four, three, two and one component waves travel along conductors 1, 2, 3, and 4, respectively. The component wave which is present on all four conductors is highly attenuated and is not discernible in the figures.

conductors at 5.77  $\mu s$  is highly attenuated within a short distance from the origin. This wave is not discernible in the figures.

The primary voltage wave on conductor 3 (stainless steel) attenuates significantly with time. With increase in distance along the cable, the peak value of this wave increases, although the attenuation rate with time increases also. In general, the front time of the component waves increases with distance along the cable.

The voltage difference between each pair of conductors can also be ascertained from Fig. 3. At x=1 km (Fig. 3b), no voltage difference appears between conductors 1, 2, and 3 until the second wave arrives on conductors 1 and 2 at 3.86  $\mu$ s. Conductors 1 and 2 remain at equal voltages until the fourth wave arrives on conductor 1 at 6.93  $\mu$ s, when about 0.65 p.u. is developed across conductors 1 and 2. The voltage spike developed across the cryogenic enclosure (hetween conductors 3 and 4) at x=1 km is about 0.20 p.u. For a 100-kV cable (BIL = 250 kV), 50 kV of transient voltage will thus be impressed across the cryogenic enclosure. About 0.24 p.u. (i.e. 59.5 kV) will be impressed across the helium space between conductors 2 and 3. The major dielectric (between conductors 1 and 2) will carry only 0.65 p.u. (i.e. 162.5 kV).

The voltage stress across the cryogenic enclosure can be relieved by reducing the BIL (hence the steady-state voltage rating) of the cable. Other alternative solutions such as metallic shorts across the enclosure at regular intervals and design optimization in the cable and system can also reduce this voltage stress. Parametric changes in the cable and system are discussed in the companion paper [3].

# Limitations of the Analysis

The  $\delta$ -matrices were evaluated by considering only the first two terms of the Eqs. (1) and (2) in order to keep the analysis simple and manageable. As a consequence, this analysis is valid at the wavefront of transients. To provide definition to the valid time duration, extensive computations were carried out by comparing results from the first terms of Eqs. (1) and (2), with those from the first two terms.

It was found that one of the eigenvalues  $(q_{3m})$  of the B<sub>2</sub> matrix in Eq. (8) was negative. This will increase the corresponding exponential term in Eq. (10) with increase in distance along the cable. It was also found that all but one resultant matrix,  $[a_1k] \times [a_2k] \times [a_3m]$ , which is multiplied by this exponential term, is null. It was essential that this particular matrix is non-zero so that the initial conditions at x=0 are consistent. This term was evaluated by using the exponential equivalent of erfc(y) for large values of y, i.e.,

$$erfc(y) \simeq exp(-y^2)/y\sqrt{\pi}$$
, (16)

where

$$y = q_{2x}^{x/4\sqrt{t-q_{1K}x}} .$$

The time limit was set such that  $y^2 > q_{3m} x/2$ . By simplifying the algebra, one gets

$$(t-t_{1k}) < q_2^2 \ell / 8q_{3m}, \tag{17}$$

where

t = time limit  $q_{1K} = 1/q_{1K}$ , and  $q_{3m}$  are the eigenvalues corresponding to the non-zero matrix  $[a_{1K}]x$   $[a_{2k}]x[a_{3m}]$ .

More terms of Eqs. (1) and (2) should be included if computation at longer times is desired. As the front time is of concern in most applications, inclusion of just the first two terms of Eqs. (1) and (2) should be sufficient, considering the elegance, simplicity, and economy in computation time of the method described.

#### CONCLUSIONS

- The step response of a dc superconducting cable to transient voltages has been described using the Laplace transformation technique, taking into account the transient impedances of the cable conductors and earth.
- (2) Transient voltages will not stress the major dielectric of the cable system to its full BlL capability.
- (3) Surge protection, cable design optimization, and system coordination will be required to limit transient voltages across the cryogenic enclosure of the cable.

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